MECHANICS OF MACHINES I LABORATORY EXERCISES

I. MOMENT OF INERTIA

It is often necessary to know the moment of inertia of some part of a machine. If that part is available, it is usually simplest to determine it experimentally rather than calculate it.
The choice of methods available may be any of the following:

1. Torsional Pendulum (Trifilar suspension).
2. Compound Pendulum.
3. Calculational method.

OBJECT OF EXPERIMENTS

To determine the moment of inertia by methods (i) and (ii) and to compare them with method (iii)

(i) <u>TORSIONAL PENDULUM</u>

Theory: It can be shown that a test piece will oscillate with simple harmonic motion when placed upon a platform suspended by wires and give a torsional displacement giving..................

$$P = 2\pi\sqrt{\frac{I_g\, l}{gmr^2}}$$

Where P = time period per oscillation,
I_g= moment of inertia about centre of gravity,
l = Length of wires,
g = Gravitation acceleration (9.81m/sec)
m= mass (Kg)
r = radius at which wires are fixed to platform

EXPERIMENTAL METHOD

Set platform oscillating and time a set number of oscillations (20). Remember to keep the oscillations small. Place test specimen on platform with its centre of gravity over centre of gravity of platform. Retime period for the same set number of oscillations. Repeat test with centre of gravity of specimen 0.1 meter from centre of gravity of platform. Check the effect of placing the specimen radially and tangentially to the platform's axis.

(ii) <u>COMPOUND PENDULUM</u>

It can be shown that the period for simple harmonic motion of a compound pendulum is given by

$$P = 2\pi\sqrt{\frac{I_g + ma^2}{m \bullet g \bullet a.}}$$

Where ***p***, ***m*** and ***g*** are as before and a **=** distance of centre of gravity of specimen from point suspension.

<u>EXPERIMENTAL METHOD</u>

Suspend the specimen with the bore of the small end of a knife-edge. Time the period for a set number of oscillations. Repeat by suspending specimen with the bore of the large end, of a knife-edge.

(iii) <u>CALCULATIONS</u>

The results from the two previous experiments may now be compared with the following calculations results. These results are obtained by considering each individual component shape of the specimen. Calculated results are based upon the following standard formula for moments of inertia about the centre of gravity whose axis is perpendicular to the plane of rotation. Moments of inertia about an axis parallel to the axis of the c use the parallel axis Theorem.

Moment of inertia of rectangular section $= m \quad \frac{a^2}{12} + \frac{b^2}{12} \qquad \frac{l^2}{2}$

Moment of inertia of rod section $= \frac{mr^2}{2} \quad \frac{r^2}{2}$

Moment of inertia of circular section $= m\left(\frac{r^2}{4} + \frac{l^2}{12}\right)$

Parallel axis theorem $Ia = I + mx^2$

Where x = displacement of axis from c of g

Total moment of inertia of specimen about the centre of gravity.

(0.0140kgm^2 = Specimen A
(0.0174 kgm^2= Specimen B

GENERAL CONCLUSIONS
Compare experimental results with calculated result. Compare the methods (i) (ii) and (iii) and comment upon the relative ease and accuracy of the various methods. Indicate possible sources of error.

2. MOMENT OF INERTIA OF A FLY WHEEL

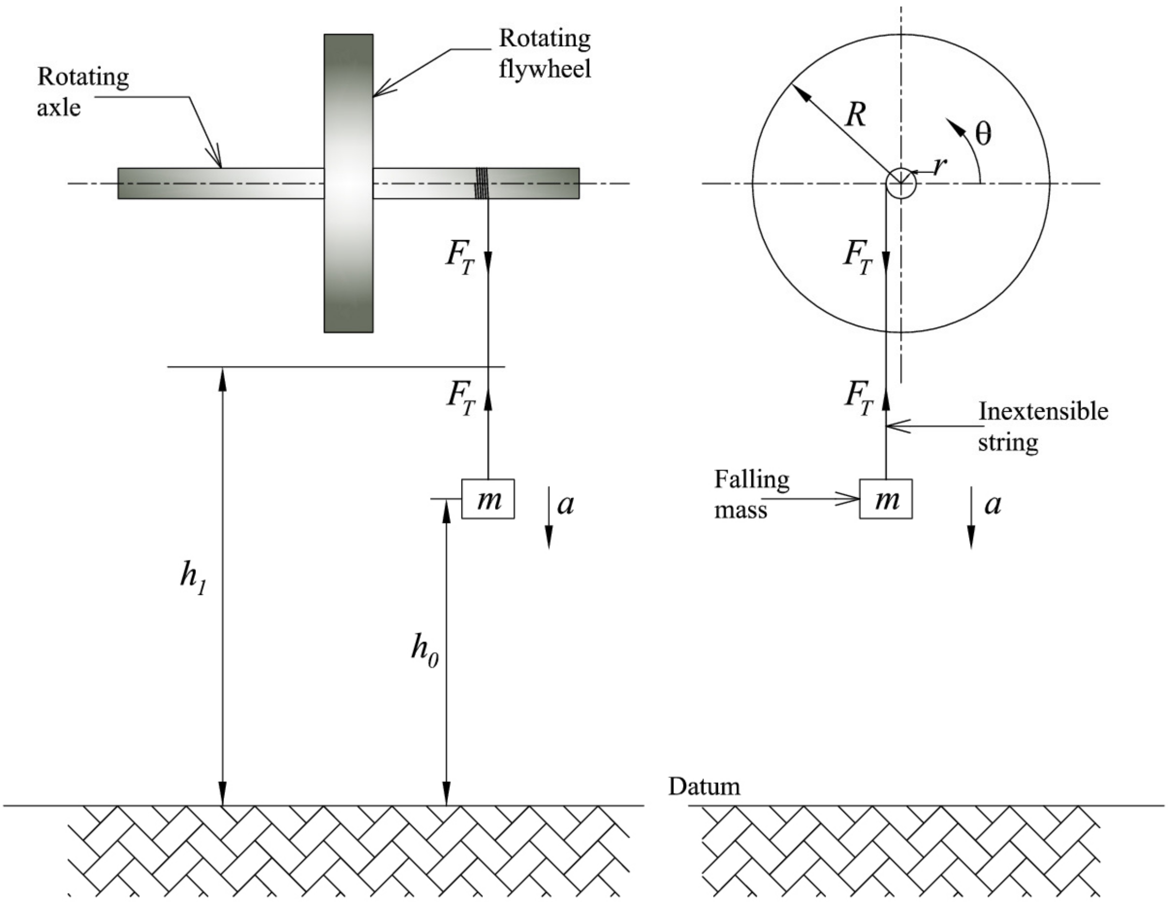
OBJECT: To determine the moment of inertia of a flywheel using the work-energy and the force-inertia equations.

APPARATUS

A flywheel with mass ***M*** and moment of inertia ***I*** is mounted horizontally in ball bearings.

A string with a mass ***m*** can be wound around the shaft. When the mass m changes height (h) the flywheel rotates (θ)

$gmh = h1 - h0$



METHOD:

- Measure and record the radii of the flywheel and the axle (***R, r***) the mass of the flywheel (***M***) and the mass to be hung from the string (***m***).
- Measure and record the lowest height of the hung mass (h_0) which the string will remain attached to the axle
- Wind the string through 2 to 5 complete revolutions (θ_1) around the axle and then measure and record the new height h_1.
- Release the system, allowing the mass to fall and rotate the flywheel and axle. As you release the system, use a stop watch to measure and record the duration from the moment the system is released to the moment the string falls off the axle (t_1), as well as the duration from the moment the system is released to the moment the flywheel finally stops rotating (t_2). Measure and record too, the number of revolutions (θ_2) – including fractions- that the flywheel rotates through from the moment the system is released to the moment the flywheel finally stops rotating.
- Repeat the experiment to check measured values.
- Repeat the experiment with a larger value of **(θ)**
- Repeat the experiment with a larger mass (***m***).

CALCULATIONS:

- Establish energy and acceleration equations for the system.
- Compute the moment of inertia (***I***) for the flywheel.

HINTS:

See that angular accelerations θ are constants and use this together with t and θ.

The bearings resistance due to friction is the product of gravitational force and frictional coefficient and the bearing radius. (in this case point bearings assumed frictionless).

RESULTS:

- Present the experiment, the equations and calculations in a report.
- Discuss your results and main sources of errors.
- Compute the moment of inertia from $I = \sum mr^2$ and compare the results.
- Draw your conclusions

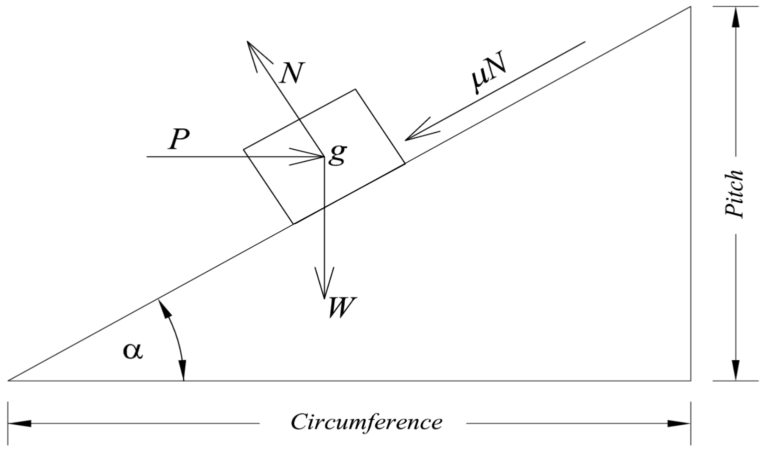
Mass	θ	T_1	T_2	***No. of Revolutions***
1.	**2** πD			
	3 πD			
	4 πD			
2.	**2** πD			
	3 πD			
	4 πD			
3.	**2** πD			
	3 πD			
	4 πD			

REFERENCE: *Hannah and Stephens: Mechanics of Machines Chap. 1.*

3. THE SCREW JACK

A screw thread can be regarded as an inclined plane wrapped round a cylinder. The shape of the plane being found from:

$$\text{Tan } \alpha = \frac{Pitch}{\pi \, x \; Mean\ diameter}$$



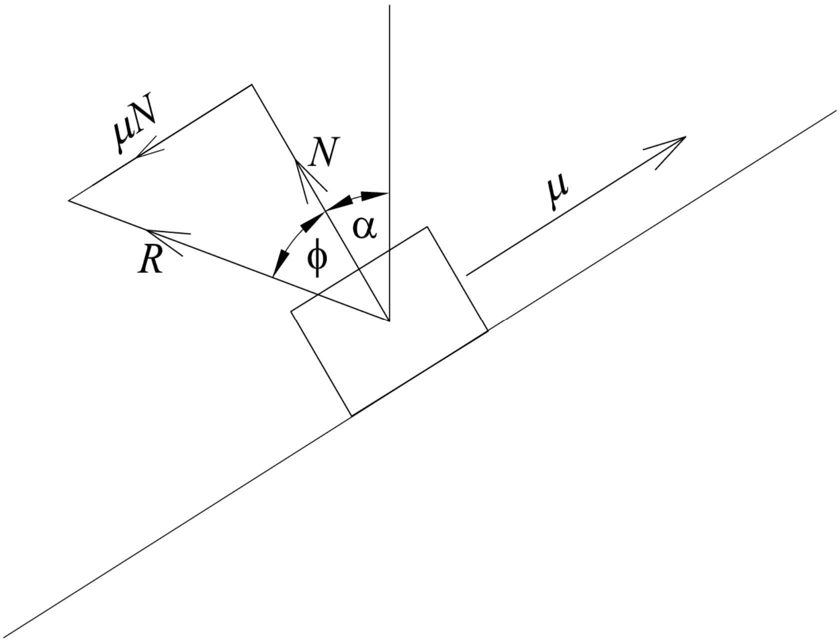
If a weight **W** is forced up the planed by a horizontal force **P**, it will be resisted by the force of friction **μN** where n is the normal reaction of the plane. When the weight starts to move up the plane there will be a force **μN** down the plane and a reaction **N** normal to the plane. The resultant **R** will be inclined at the friction angle α to the normal. The friction angle is defined as tan φ = μ
This reaction **R** will be the resultant of the horizontal force **P** and the weight **W**

$$\therefore \quad P = W \tan(\alpha + \varphi) \text{ ……………………….. (i)}$$

$$= W \frac{\tan\alpha + \tan\phi}{1 - \tan\mu \tan\phi}$$

But tan φ = μ

$$P = W \left(\frac{\tan\alpha + \mu}{1 - \mu \tan\alpha} \right) \text{………………………………….. (ii)}$$



In the case of a screw jack the tongue required to turn the nut T = $\left(P.\frac{d}{2}\right)$, where ***d*** is the mean diameter of the screw.

Substitution for P from equation (ii)

$$T = \frac{Wd}{2}\left(\frac{\tan\alpha + \mu}{1 - \mu\tan\alpha}\right) \text{.. (iii)}$$

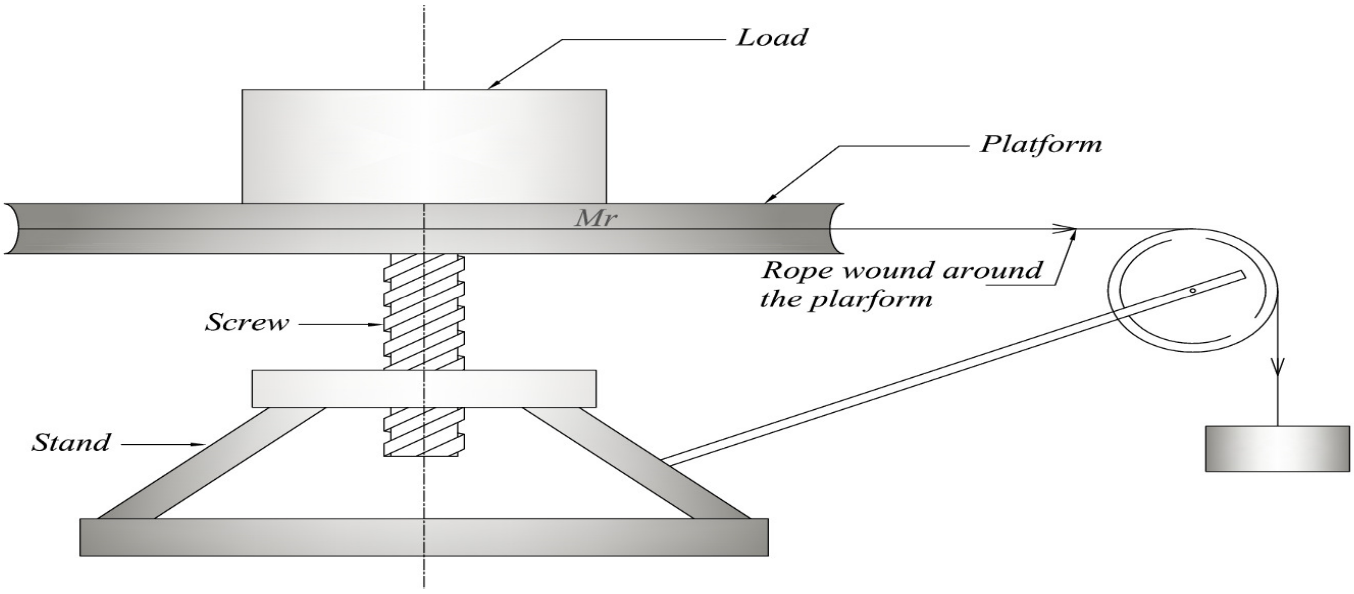
For lowering a load the expression will be altered by the change of direction of the friction angle

$$T = \frac{Wd}{2}\left(\frac{\tan\alpha - \mu}{1 + \mu\tan\mu}\right) \text{.. (iv)}$$

If μ is less than tan α, T will be positive, that is in the same direction as for lifting. This means that unless the screw is restrained, it will lower the load by itself.

OBJECT: To determined the efficiency of a screw jack, and the coefficient of friction between Screwthreads.

APPARATUS: Screw Jack fitted with a weight platform and a means of applying a known torque to the screw.



METHOD: Measure the pitch and mean diameter of the screw, and the radius of the rope drum.

Place loads from 2 to 20 kg. on the platform and determine the torque required to just raise the load. Repeat for lowering the load.

NOTE: Consider what you should do to compensate for the torque required to turn pulley A in this experiment. Make the necessary measurements to determine this correction.

RESULTS: The figures for load Vs torque should be plotted as a graph together with the efficiency η.

$$\eta = \frac{Workout}{Workin} = \frac{LoadxPitch}{2\pi\chi Torgue}$$

The coefficient of friction μ for both raising and lowering the load should be calculated from the equation.

$$\mu = \frac{2T/W - d\tan\alpha}{2\frac{T}{W}\tan\alpha + d}$$ ……………… for raising

and $$\mu = \frac{d\tan\alpha - 2\frac{T}{W}}{2\frac{T}{W}\tan\alpha + d}$$ ……………for lowering

CONCLUSIONS: Discuss the results, and indicate possible sources of error.

4. BELT FRICTION

OBJECT: To determine the coefficient of friction between belt and pulley and to check the effect of lap angle on the grip of the belt

APPARATUS:

See Fig. I. The belt friction rig consists of a pulley ***A,*** free to rotate about the spindle ***B***. The arm ***C*** can be adjusted to give various lap angles to the belt under test.

The belt ***D*** is connected to the spring balance ***E*** attached to the arm ***C***. A weight carrier is hung on the other end of the belt, which hangs over the pulley wheel.

METHOD:

Arm ***C*** is set to the required lap angle and a weight added to the weight carrier (T_2). The pulley wheel is then twisted by hand in the direction that will increase the reading on the spring balance (T_1). The static friction coefficient can be determined from the value of T_1 just before the belt slips. Further weights should be added to the carrier until the spring balance reads approximately 25kgf. Weights should then be removed from the carrier so as to give 5-7 equally space reading between 0 and 25 kgf for T_1.

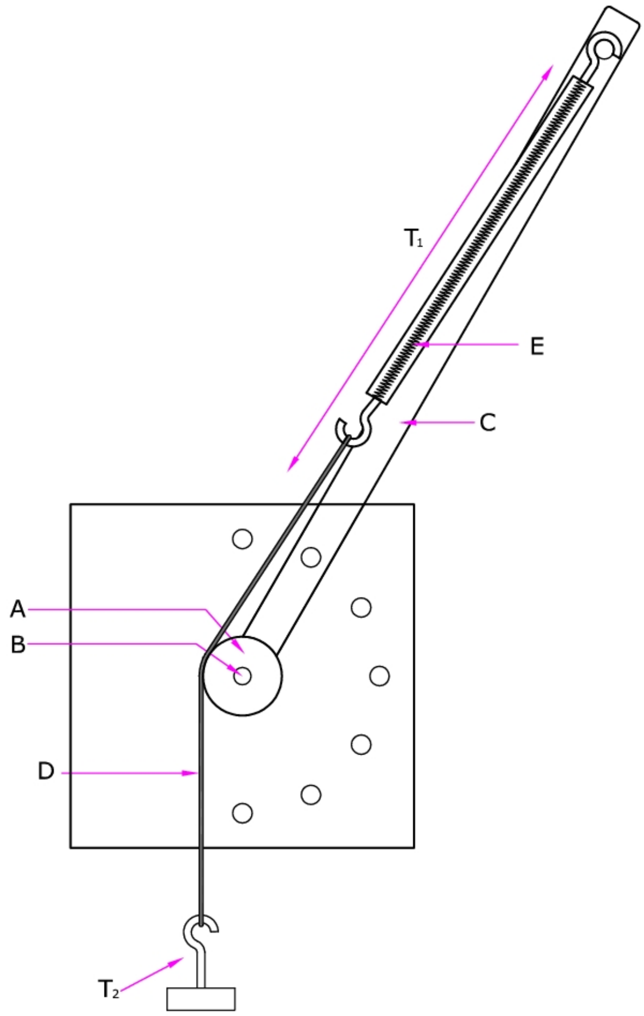


Figure 1

(*For your calculations **1** 1bf =**4.45** N)

THEORY:

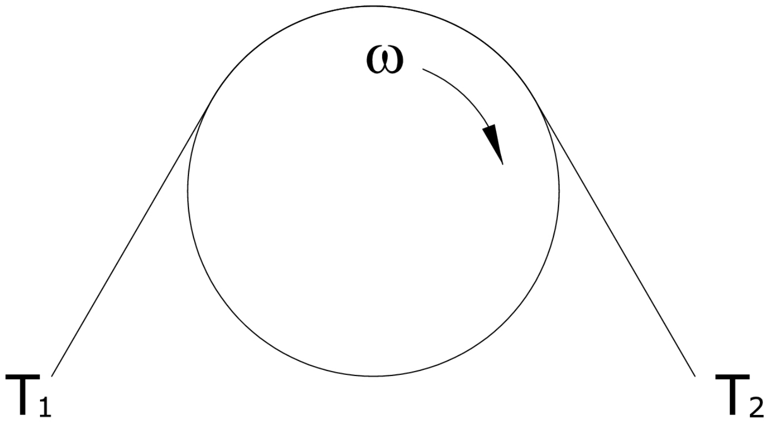


Figure 2

A flat belt is wrapped round a segment of a flat pulley. The limiting tensions in the two ends of the belt are T_1 and $T_{2.}$ Refer Fig ii

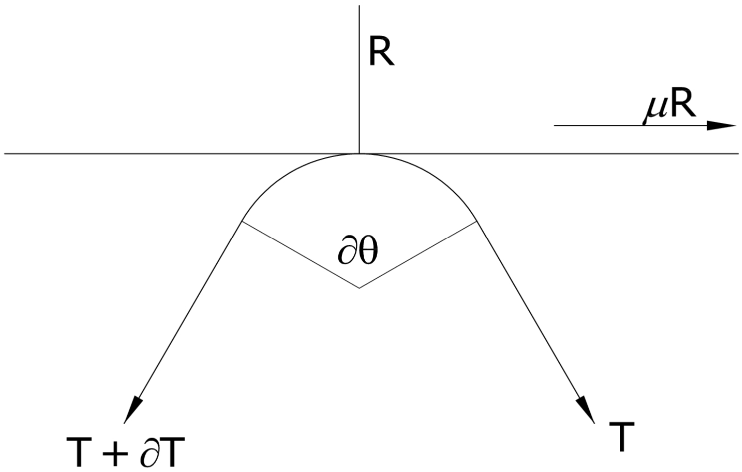


Figure 3

Consider a small arch of the pulley ∂. The tension on one side will be ***T*** and the other side ***T+*** ∂ ***T.*** Let the radial reaction that ***CI*** the element of the belt be ***R***. then for the limiting case the frictional force = R.

Resolving forces radially. *R*= *(T+ dT) sin* $\frac{\partial\theta}{2} + T\sin\frac{\partial\theta}{2}$

Neglecting 2nd order of small quantities and writing $Sin\,\partial\theta = \partial\theta,$
R= *T* $\partial\,\theta$ ………………………………………………...………(i)
Then resolving forces tangentially ∂ *T* = *R.* ………………………(ii)

From (i) and (ii) ∂ *T* = *T* $\partial\,\theta$ *or* $\frac{\partial \mathrm{T}}{\mathrm{T}} = a\partial\theta$

Integrating $\int_{T_2}^{T_1}\frac{\partial T}{T} = \mu\partial\theta$

i.e. $T_1 = \mu\theta$ ……………..(iii)

Note that this equation holds only when a slip is impending.

Refer to Fig IV

For a vee-belt the normal reaction between belt and pulley = N

$\therefore$ $R = 2N \text{ cosec } (\frac{\beta}{2})$, where β is vee or groove angle.

Then for the limiting case,

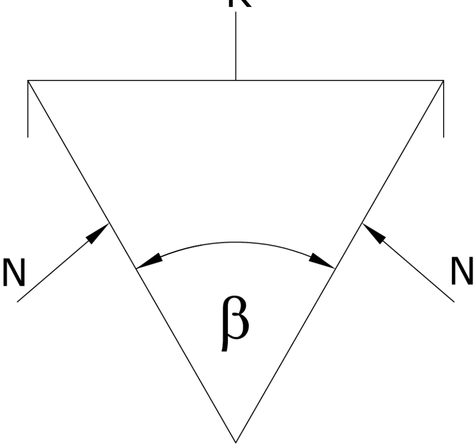


Figure 4

Frictional force = $2\mu N$

$\mu R \text{ cosec } \frac{\beta}{2}$ …………………..(iv)

$\partial T = \mu R \text{ cosec } (\frac{\beta}{2})$…………..(v)

Equation (v), in conjunction with eq.(i) which still holds finally yields, on integration

$$\frac{T_1}{T_2} = e^{\mu \beta \text{ cosec } (\frac{\beta}{2})} \quad \text{.............(vi)}$$

Experimental Results

Graph of T_l against T_2 should be obtained for lap angles of *30,90,135* and 180 each curve consisting of 5-7 equally spaced points to give values of T_2 between 0-25 kgf.

From the best straight line fit of the points the ration $\frac{T1}{T2}$ found, and the value for μ calculated.

Conclusions

Comment o n the accuracy of your results and give your ideas as to why there should be any variation in the value of μ as found at the different lap angles.